16. Given that the lengths of three sides, a, b, c of a triangle are positive integers, where $a < b < c$. Find the number of the triangles with $b = 2$.

17. In a right-angled triangle, if the length of a leg is 21, and the lengths of the other two sides are also positive integers, find the minimum value of its possible perimeter.

18. In right-angled ΔABC, $\angle C = 90°$, E is on BC such that AC = BE. D is on AB such that $DE \perp BC$. Given that $DE + BC = 1$, $BD = \frac{1}{2}$, find $\angle B$ in degrees.

19. In ΔABC, AC = BC, $\angle C = 20°$, M is on the side AC and N is on the side BC, such that $\angle BAN = 50°$, $\angle ABM = 60°$. Find $\angle NMB$ in degrees.

20. In the ΔABC, BE is the angle bisector of the $\angle ABC$, AD is the median on the side BC, and AD intersects BE at O perpendicularly. Given BE = AD = 4, find the lengths of three sides of ΔABC.

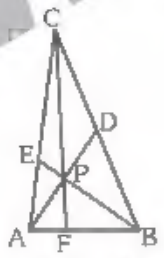
21. In ΔABC, D is the midpoint of BC, E is on AC such that AC = 3EC. BE and AD intersect at G. Find AG : GD.

22. When extending the sides AB, BC, CA of ΔABC to B', C', A' respectively, such that AB' = 2AB, CC' = 2BC, AA' = 3CA. If area of ΔABC is 1, find the area of $\Delta A'B'C'$.

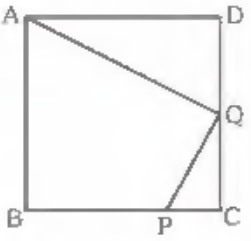
23. Given that the point P is outside the equilateral triangle ABC but inside the region of $\angle ABC$. If the distances from P to BC, CA, AB are h_1, h_2 and h_3 respectively, and $h_1 - h_2 + h_3 = 6$, find the area of ΔABC.

24. Given that G is the centroid of ΔABC, $GA = 2\sqrt{3}$, $GB = 2\sqrt{2}$, GC = 2. Find the area of ΔABC.

25. Point P is inside ΔABC. Line segments APD, BPE, and CPF are drawn with D on BC, E on CA, and F on AB (see the figure below). Given that AP = 6, BP = 9, PD = 6, PE = 3, and CF = 20, find the area of ΔABC.

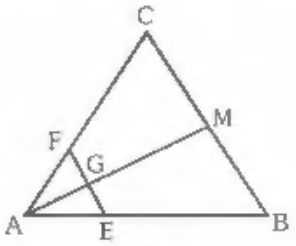


26. In a square ABCD, let P be a point on the side BC such that BP = 3PC and Q be the mid-point of CD. If the area of the triangle PCQ is 5, what is the area of triangle QDA?



27. A triangle ΔABC is inscribed in a circle of radius 4 cm. Suppose that $\angle A = 60°$, AC – AB = 4 cm, and the area of ΔABC is x cm^2. Find the value of $\left(\frac{x}{2}\right)^2$.

28. In ΔABC, $AB : AC = 4 : 3$ and M is the midpoint of BC. E is a point on AB and F is a point on AC such that $AE : AF = 2 : 1$. It is also given that EF and AM intersect at G with $GF = 36$ cm and $GE = x$ cm. Find the value of x.



29. Nondegenerate ΔABC has integer side lengths, BD is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

30. It is known that the 3 sides of a triangle are consecutive positive integers and the largest angle is twice the smallest angle. Find the perimeter of this triangle.

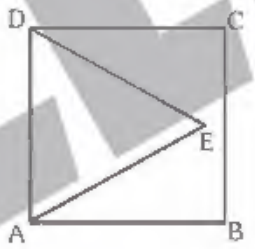
31. A triangle ΔABC is inscribed in a circle of radius 1, with $\angle BAC = 60°$. Altitudes AD and BE of ΔABC intersect at H. Find the smallest possible value of the length of the segment AH.

32. In triangle ABC, $AB = 28$, $BC = 21$ and $CA = 14$. Points D and E are on AB with $AD = 7$ and $\angle ACD = \angle BCE$. Find the length of BE.

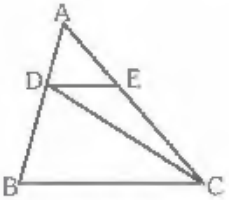
33. Four points in the order A, B, C, D lie on a circle with the extension of AB meeting the extension of DC at E and the extension of AD meeting the extension of BC at F. Let EP and FQ be tangents to this circle with points of tangency P and Q respectively. Suppose $EP = 60$ and $FQ = 63$. Determine the length of EF.

34. The diagram shows an equilateral triangle ADE inside a square ABCD. What is the value of

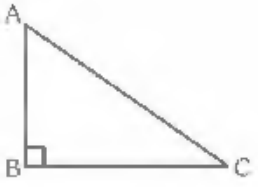
$$\left(\frac{\text{area of } \Delta ADE}{\text{area of } \Delta DEC}\right)^2$$



35. In this question, $S_{\Delta XYZ}$ denotes the area of Δ_{XYZ}. In the following figure, if $DE \parallel BC$, $S_{\Delta ADE} = 1$ and $S_{\Delta ADC} = 4$, find $S_{\Delta DBC}$.



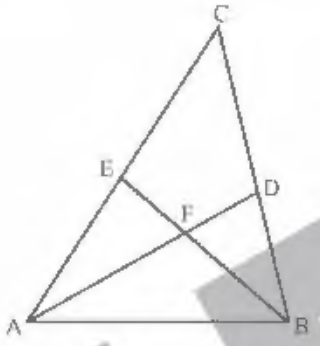
36. ΔABC is a right-angled triangle with $\angle ABC = 90°$. A circle C_1 is drawn with AB as diameter, and another circle C_2 is drawn with BC as diameter. The circles C_1 and C_2 meet at the points B and P. If $AB = 5$ cm, $BC = 12$ cm and $BP = x$ cm, find the value of $\frac{120}{x}$.



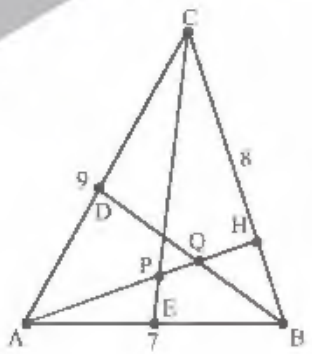
59. In ΔPAT, $\angle P = 36°$, $\angle A = 56°$, and $PA = 10$. Points U and G lie on sides $\overline{TP}$ and $\overline{TA}$, respectively, so that $PU = AG = 1$. Let M and N be the midpoints of segments $\overline{PA}$ and $\overline{UG}$, respectively. What is the degree measure of the acute angle formed by lines MN and PA?

60. Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. The ratio of the area of $\Delta A'B'C'$ to the area of ΔABC is Q : 1. What is Q ?

61. In ΔABC, $AB = 6$, $BC = 7$, and $CA = 8$. Point D lies on $\overline{BC}$, and $\overline{AD}$ bisects $\angle BAC$. Point E lies on $\overline{AC}$, and $\overline{BE}$ bisects $\angle ABC$. The bisectors intersect at F. The ratio of AF : FD is m: n. What is $m^2 + n^2$.



62. In ΔABC shown in the figure, $AB = 7$, $BC = 8$, $CA = 9$, and $\overline{AH}$ is an altitude. Points D and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $\overline{BD}$ and $\overline{CE}$ are angle bisectors, intersecting $\overline{AH}$ at Q and P, respectively. If $PQ = \frac{a}{b}\sqrt{c}$, where a, b are co-prime, then what is the value of $(a + b + c^2)$?



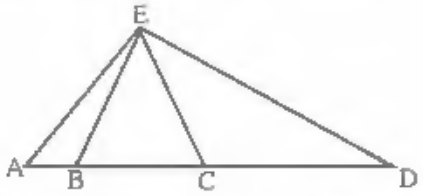
63. There are f noncongruent integer-sided triangles with positive area and perimeter less than 15, which are neither equilateral, isosceles, nor right triangles. What is f ?

64. In ΔBAC, $\angle BAC = 40°$, $AB = 10$, and $AC = 6$. Points D and E lie on $\overline{AB}$ and $\overline{AC}$ respectively. What is the minimum possible value of $BE + DE + CD$?

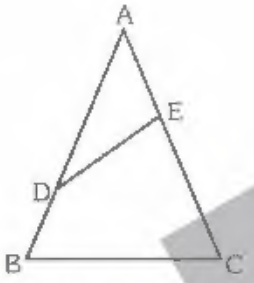
65. In ΔABC, $AB = AC = 28$ and $BC = 20$. Points D, E and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively.What is the perimeter of parallelogram ADEF?

73. In triangle ABC, AB = $(b^2 - 1)$ cm, BC = a^2 cm and AC = 2a cm, where a and b are positive integers greater than 1. Find the value of a – b.

74. In the figure below, ADE is a triangle with ∠AED = 120°, and B and C are points on side AD such that BCE is an equilateral triangle. If AB = 4 cm, CD = 16 cm and BC = x cm, find the value of x.



75. The figure below shows a triangle ABC where AB = AC. D and E are points on sides AB and AC, respectively, such that AB = 4DB and AC = 4AE. If the area of the quadrilateral BCED is 52 cm^2 and the area of the triangle ADE is x cm^2, find x.



76. In the triangle ABC, AC = 2BC, ∠C = 90° and D is the foot of the altitude from C onto AB. A circle with diameter AD intersects the segment AC at E. Find AE : EC.

77. In the triangle ABC, AB = 8, BC = 7 and CA = 6. Let E be the point on BC such that ∠BAE = 3∠EAC. Find

$$\frac{4AE^2}{5}.$$

78. In the triangle ABC, the bisectors of ∠A and ∠B meet at the incentre I, the extension of AI meets the circumcircle of triangle ABC at D. Let P be the foot of the perpendicular from B onto AD, and Q a point on the extension of AD such that ID = DQ. Determine the value of (BQ × IB)/(BP × ID).

79. Three sides OAB, OAC and OBC of a tetrahedron OABC are right-angled triangles, i.e. ∠AOB = ∠AOC= ∠BOC = 90°. Given that OA = 7, OB = 2 and OC = 6, find the value of

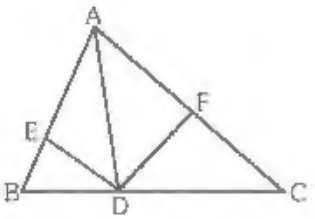
[(Area of ΔOAB)² + (Area of ΔOAC)² + (Area of ΔOBC)² + (Area of ΔABC)² + 37]$^{1/2}$.

80. In a triangle ABC, the length of the altitudes AD and BE are 4 and 12 respectively.

Find the largest possible integer value for the length of the third altitude CF.

81. In the figure below, ABC is a triangle and D is a point on side BC Point E is on side AB such that DE is the angle bisector of ∠ADB, and point F is on side AC such that DF is the angle bisector of ∠ADC. Find the value of

$$\frac{AE}{EB}\cdot\frac{BD}{DC}\cdot\frac{CF}{FA}.$$



127. Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. If the ratio of the area of $\Delta A'B'C'$ to the area of ΔABC is $x : 1$. Find the value of x.

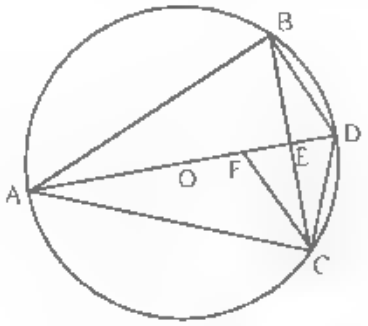
128. A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 gm. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. If W is the weight, in gm, of the second piece. Then find [W], where [x] denotes the greatest integer less than or equal to x.

129. The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. If the area of ΔCDE can be written as $\frac{a(2-\sqrt{b})}{2}$. Find the product of a and b.

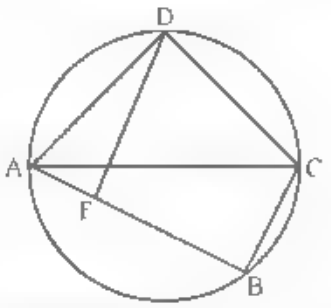
Important Notes

83. A circle with center O has area 156π. Triangle ABC is equilateral. $\overline{BC}$ is a chord on the circle. $OA = 4\sqrt{3}$ and point O is outside ΔABC. What is the side length of ΔABC?

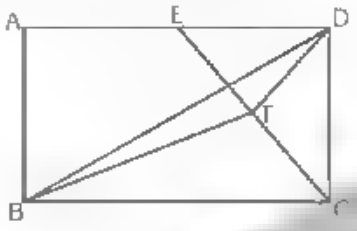
84. In the figure below, ABC is an isosceles triangle inscribed in a circle with centre O and diameter AD, with AB = AC. AD intersects BC at E. and F is the midpoint of OE. Given that BD is parallel to FC and BC = $2\sqrt{5}$ cm, find square of the length CD (in cm)



43. Let ABCD be a quadrilateral inscribed in a circle with diameter AC and let E be the foot of perpendicular from D onto AB as shown in the figure below. If AD = DC and the area of quadrilateral ABCD is 24 cm² find the square of length of DE (in cm)



44. In the figure below, ABCD is a rectangle E is the midpoint of AD and F is the midpoint of CE. If the area of triangle BDF is 12 cm², find the area of rectangle ABCD in cm².

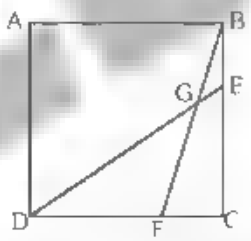


45. Given that ABCD is a square. Points E and F lie on the side BC and CD respectively, such that $BE = CF = \frac{1}{3}AB$

G is the intersection of BF and DE. If

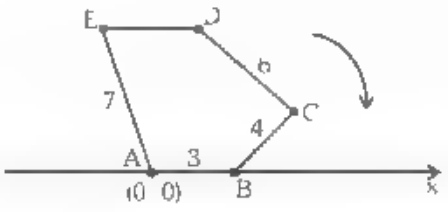
$$\frac{\text{Area of } \Delta BGD}{\text{Area of } ABCD} = \frac{m}{n}$$

is in its lowest term, find the value of m + n



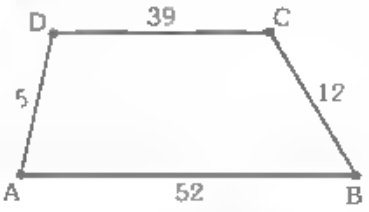
46. Rectangle ABCD has AB = 4 and BC = 3. Segment EF is constructed through B so that EF is perpendicular to DB and A and C lie on DE and DF, respectively. What is EF to the nearest integer?

47. As shown below, convex pentagon ABCDE has sides AB = 3, BC = 4, CD = 6, DE = 3, and EA = 7. The pentagon is originally positioned in the plane with vertex A at the origin and vertex B on the positive x-axis. The pentagon is then rolled clockwise to the right along the x-axis. What is the length side which will touch the point x = 2009 on the x-axis?



48. Rectangle ABCD has AB = 8 and BC = 6. Point M is the midpoint of diagonal $\overline{AC}$, and E is on $\overline{AB}$ with $\overline{ME} \perp \overline{AC}$. What is the area of ΔAME to the nearest integer?

64. In trapezoid ABCD with bases AB and CD, we have AB = 52, BC = 12, CD = 39, and DA = 5. The area of ABCD is given by 10A. Find the value of A.



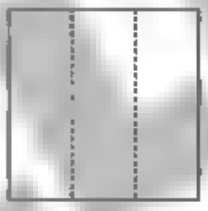
65. A regular octagon ABCDEFGH has sides of length two. Then the area of ΔADG is given by $(a + b\sqrt{c})$. Find the product of a, b & c.

66. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 20. Let the length of each side of the octagon is given by $a(\sqrt{b} - c)$. Find the product of a, b and c.

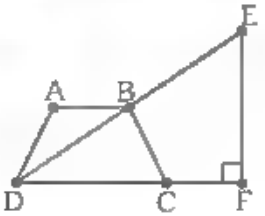
67. A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Then the radius of the cylinder is R. Find the value of (11R).

68. In trapezoid ABCD, $\overline{AB}$ and $\overline{CD}$ are perpendicular to $\overline{AD}$ with AB + CD = BC, AB < CD, and AD = 7. What is (4AB · CD)?

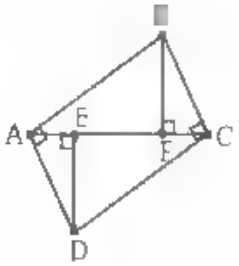
69. A square is cut into three rectangles along two lines parallel to a side, as shown. If the perimeter of each of the three reactangles is 24, then the area of the original square is



70. In the figure, ABCD is an isosceles trapezoid with side lengths AD = BC = 5, AB = 4, and DC = 10. The point C is on $\overline{DF}$ and B is the midpoint of hypotenuse $\overline{DE}$ in the right triangle DEF. Then CF =

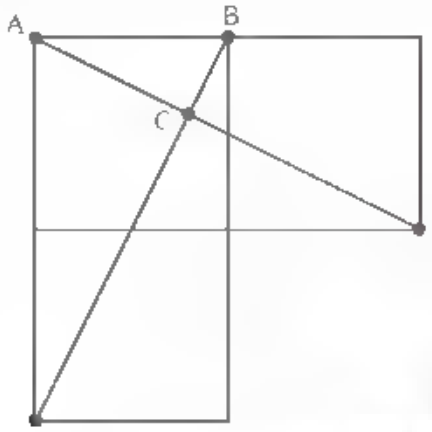


71. In the figure, ABCD is a quadrilateral with right angles at A and C. Points E and F are on $\overline{AC}$, and $\overline{DE}$ and $\overline{BF}$ are perpendicular to $\overline{AC}$. If AE = 3, DE = 5, and CE = 7, then the value of $BF - \frac{1}{5}$ is



92. A wire is cut into two pieces, one of length a and the other of length b. The piece of length a is bent to form an equilateral triangle and the piece of length b is bent to form a regular hexagon. The triangle and the hexagon have equal area. What is $\frac{4a^2}{b^2}$

93. Three unit squares and two line segments connecting two pairs of vertices are shown. If the area of ΔABC is N, find 5N.



94. In rectangle ABCD, AB = 6, AD = 30, and G is the midpoint of $\overline{AD}$. Segment AB is extended 2 units beyond B to point E, and F is the intersection of $\overline{ED}$ and $\overline{BC}$. If the area of BFDG is X, find |X|.

95. Square EFGH has one vertex on each side of square ABCD. Point E is on $\overline{AB}$ with AE = 7 EB. If the ratio of the area of EFGH to the area of ABCD is $\frac{m}{n}$, where m, n are co-prime, find m + n.

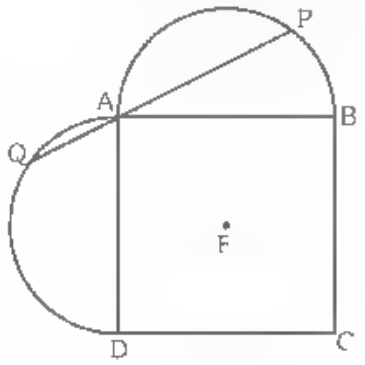
96. Rectangle ABCD has AB = 6 and BC = 3. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

97. Points A and C lie on a circle centered at O, each of $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are tangent to the circle, and ΔABC is equilateral. The circle intersects $\overline{BO}$ at D. What is BD + BO ?

98. Suresh has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

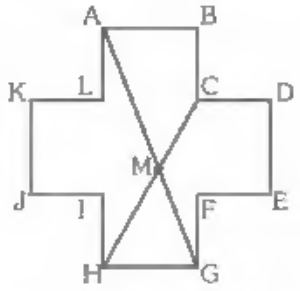
99. Equiangular hexagon ABCDEF has side lengths AB = CD = EF = 1 and BC = DE = FA = r. The area of ΔACE is 70% of the area of the hexagon. What is the sum of all possible values of r?

100. In the following diagram, ABCD is a square, and E is the center of the square ABCD. P is a point on a semi-circle with diameter AB. Q is a point on a semi-circle with diameter AD. Moreover, Q, A and P are collinear (that is, they are on the same line). Suppose QA = 14 cm, AP = 46 cm, and AE = x cm. Find the value of x.



118. Convex quadrilateral ABCD has AB = 9 and CD = 12. Diagonals AC and BD intersect at E, AC = 14, and ΔAED and ΔBEC have equal areas. What is AE?

119. Consider the 12-sided polygon ABCDEFGHIJKL, as shown. Each of its sides has length 4, and each two consecutive sides form a right angle. Suppose that $\overline{AG}$ and $\overline{CH}$ meet at M. What is the area of quadrilateral ABCM to the nearest integer?



120. A paint brush is swept along both diagonals of a square to produce the symmetric painted area, as shown. Half the area of the square is painted. If the ratio of the side length of the square to the brush width is $m\sqrt{2} + n$ then find the value of $(m^2n)^2$?

ANSWER KEY

EXERCISE - 4.1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	3	4	2	1	2	4	3	1	2	1	1	3	1	1

15. 8 **16.** 0 **17.** 70 **19.** 30 **20.** $AC = 3\sqrt{5}$, $AB = \sqrt{13}$, $BC = 2\sqrt{13}$

21. 4 : 1 **26.** 20 **27.** 48 **28.** 54 **29.** 33 **30.** 15 **31.** 01

32. 12 **33.** 87 **34.** 03 **35.** 12 **36.** 26 **37.** 30 **38.** 20

39. 01 **40.** 03 **41.** 06 **42.** 35 **43.** 20 **44.** 02 **45.** 50

46. 15 **47.** 02 **48.** 70 **49.** 77 **50.** 52 **51.** 12 **52.** 03

53. 20 **54.** 18 **55.** 14 **56.** 02 **57.** 75 **58.** 12 **59.** 80

60. 37 **61.** 05 **62.** 48 **63.** 05 **64.** 14 **65.** 56 **66.** 34

67. 36 **68.** 21 **69.** 15 **70.** 30 **71.** 10 **72.** 48 **73.** 00

74. 08 **75.** 12 **76.** 04 **77.** 27 **78.** 02 **79.** 33 **80.** 05

81. 01 **82.** 24 **83.** 21 **84.** 27 **85.** 90 **86.** 02 **87.** 03

88. 82 **89.** 03 **90.** 15 **91.** 01 **92.** 12 **93.** 56 **94.** 24

95. 48 **96.** 02 **97.** 43 **98.** 18 **99.** 03 **100.** 23 **101.** 04

102. 65 **103.** 27 **104.** 12 **105.** 05 **106.** 09 **107.** 26 **108.** 30

109. 64 **110.** 07 **111.** 10 **112.** 05 **113.** 03 **114.** 36 **115.** 40

116. 15 **117.** 12 **118.** 10 **119.** 60 **120.** 15 **121.** 13 **122.** 20

123. 75 **124.** 50 **125.** 72 **126.** 54 **127.** 37 **128.** 33 **129.** 75

EXERCISE - 4.2

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	2	1	2	1	1	4	1	3	4	4	2	1	4	3
Que.	16	17	18	19	20	21	22	23	24						
Ans.	1	2	3	1	1	2	2	1	2						

25. 20 **26.** 57 **27.** 36 **28.** 60 **29.** 04 **30.** 05 **31.** 07

32. 49 **33.** 17 **34.** 03 **35.** 53 **36.** 72 **37.** 66 **38.** 40

39. 36 **40.** 61 **41.** 30 **42.** 63 **43.** 08 **44.** 16 **45.** 54

46. 65 **47.** 60 **48.** 78 **49.** 45 **50.** 8 **51.** 35 **52.** 82

53. 10 **54.** 11 **55.** 04 **56.** 02 **57.** 23 **58.** 44 **59.** 24

60. 02 **61.** 60 **62.** 04 **63.** 05 **64.** 03 **65.** 06 **66.** 92

67. 01 **68.** 73 **69.** 02 **70.** 06 **71.** 27 **72.** 16 **73.** 03

74. 69 **75.** 19 **76.** 08 **77.** 02 **78.** 08 **79.** 12 **80.** 03

81. 02 **82.** 26 **83.** 06 **84.** 06

EXERCISE - 4.3

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	1	1	3	1	4	3	3	2	2	1	2	1	1	1

15. $\frac{12}{5}$ **16.** 54 **17.** 60 **18.** 25 **19.** 40 **20.** 12

21. 16 **22.** 19 **23.** 01 **24.** 02 **25.** 25 **26.** 30 **27.** 31

28. 16 **29.** 21 **30.** 07 **31.** 03 **32.** 50 **33.** 16 **34.** 15

35. 11 **36.** 20 **37.** 31 **38.** 24 **39.** 42 **40.** 41 **41.** 28

42. 26 **43.** 24 **44.** 96 **45.** 23 **46.** 10 **47.** 03 **48.** 09

49. 04 **50.** 19 **51.** 98 **52.** 85 **53.** 86 **54.** 06 **55.** 01

56. 04 **57.** 08 **58.** 75 **59.** 05 **60.** 02 **61.** 05 **62.** 20

63. 25 **64.** 21 **65.** 24 **66.** 40 **67.** 30 **68.** 49 **69.** 81

70. 04 **71.** 04 **72.** 07 **73.** 01 **74.** 34 **75.** 62 **76.** 09

77. 22 **78.** 98 **79.** 45 **80.** 20 **81.** 50 **82.** 64 **83.** 81

84. 48 **85.** 31 **86.** 10 **87.** 72 **88.** 06 **89.** 05 **90.** 25

91. 49 **92.** 6 **93.** 01 **94.** 67 **95.** 57 **96.** 75 **97.** 03

98. 17 **99.** 6 **100.** 34 **101.** 45 **102.** 84 **103.** 06 **104.** 05

105. 12 **106.** 08 **107.** 01 **108.** 80 **109.** 04 **110.** 25 **111.** 24

112. 02 **113.** 26 **114.** 10 **115.** 27 **116.** 27 **117.** 10 **118.** 06

119. 18 **120.** 64